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On the pseudo-steady plastic flow during the initiation of extrusion through conical dies

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SUMMARY
During the initiation of extrusion through conical dies, a short while after the specimen's front end has entered the die working area, the prevailing plastic flow might be such that the velocity and the stress deviator are already time independent while the hydrostatic pressure is still changing with time, because of its dependence on the position within the die of the advancing front end of the specimen. This pseudo-steady flow is the subject of analysis here. Frictionless die walls, absence of body forces, negligible accelerations, and rigid-perfectly plastic Mises materials are assumed. Aside from these simplifications, the analytical solution presented is exact.

1. Introduction

The continuous flow of plastic material in a conical region bounded by frictionless walls, far from the region's entry and exit, is one of the few axisymmetrical cases where the Mises equations for ideal plasticity (i.e., flow incompressibility, balance of linear momentum, St. Venant–Levy–Mises flow rule and Maxwell–Mises yield criterion [1]) have been solved without any simplifying assumptions other than the neglect of all the body forces, including those caused by accelerations. The solution is due to Hoffman and Sachs [2] and consist of a cylindrical state of stress and a spherical velocity field of the form

$$\begin{aligned} &S_{rr} = -2S_{\theta\theta} = -2S_{\phi\phi} = \tfrac{2}{3}\sigma_0, \; S_{r\theta} = S_{r\phi} = S_{\theta\phi} = 0, \\ &p = A + 2\sigma_0 \ln r + \tfrac{2}{3}\sigma_0, \; v_r = Br^{-2}, \; v_\theta = v_\phi = 0. \end{aligned} \tag{1}$$

In (1), S denotes the stress deviator, σ_0 is the yield stress in uniaxial tension, p is the hydrostatic pressure, v denotes the flow velocity, and (r, θ, ϕ) are spherical coordinates centered at the cone apex (Figure 1); A and B are constants of integration to be determined by the conditions at the entry or exit from the conical region confining the plastic flow.

Assuming that a constant volumetric rate of flow, Q, is maintained at the die entry, flow continuity requires that

$$Q = \int_0^\alpha (-v_r)\cdot 2\pi r \sin\theta r \, d\theta, \tag{2}$$

where the minus sign appears because the flow is directed toward the cone apex, in the opposite direction of r. Substitution of v_r from (1) into (2) and integration lead to an

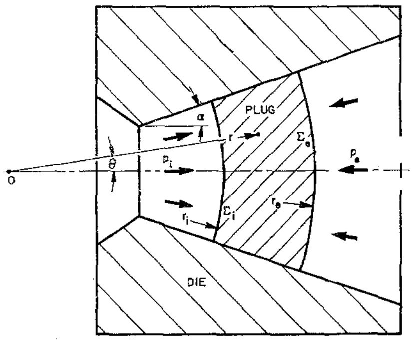

Figure 1. Plug flow geometry.

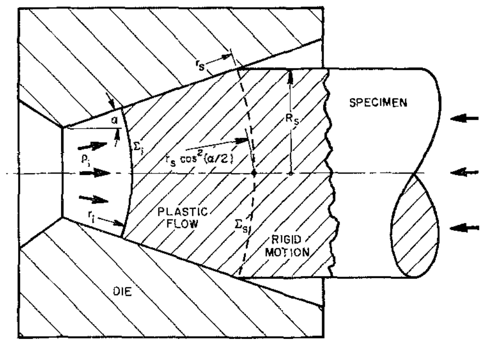

Figure 2. Initiation of extrusion.

expression for B which, when substituted back into (1), yields

$$v_r = \frac{(-1)Q}{4\pi r^2 \sin^2(\alpha/2)}. \tag{3}$$

The expression for the hydrostatic pressure, p, cannot be obtained with the same ease and generality. This is because the calculation of the constant A requires a knowledge of the exact extent of the conical region confining the plastic flow—so that a stress boundary condition can be applied at one of the limiting surfaces other than the conical one.

In their original work related to continuous metal forming with conical dies, Hoffman and Sachs assumed that: 1) the die entry and exit are both spherical surfaces centered at cone apex, and 2) the plastic flow governed by (1) prevails throughout the region bounded by the die wall—from the die entry to the die exit. Under these additional (and unjustified) assumptions, the analysis revealed that the power required to drive the process depends logarithmically on the ratio of the radius of the undeformed and the deformed specimen.

Although (1) would have seemed to be a good start for additional research into the nature of axisymmetrical solutions of Mises equations, we have not found in the literature any further attempts to calculate exactly the value of the constant of integration, A, for the problem of continuous forming, or for related problems to which the plastic flow of (1) might apply.

In this paper, we present analytical solutions to two problems which are related to metal forming through conical dies. In both cases, we find the extent of the region confining the plastic flow governed by (1), calculate the value of the constant of integration A, and then —from the solution so derived—obtain the expression for the pressure gradient required to drive each of the processes, and for the pressure distribution to be supported by the inner wall of the die.

The first problem concerns the pseudo-steady plastic flow of a short metal plug within a long conical die (Figure 1). For this problem, the analysis of Section 2 leads to a solution which fulfills the two additional assumptions made by Hoffman and Sachs mentioned earlier in this introduction. Their result, therefore, although originally presented as a solution to the problem of continuous forming, applies strictly only to the plug flow described here.

The presentation of the plug flow, aside from showing the circumstances under which Hoffman and Sach's solution is "exact" serves also as a convenient intermediate step toward the solution of the main problem in this paper, concerning the pseudo-steady plastic flow which precedes the setting in of the continuous forming by extrustion. This pseudo-steady flow is characterized by the specimen's front end being in the die working area and moving towards the die exit, while—at the same time—the back end of the specimen, still undeformed outside the die, is moving as a rigid body towards the die entry.

In Section 3, we calculate the surface which separates the rigid motion outside the die from the plastic flow inside; on this surface, located at the die entry, the velocity of the material suffers a transversal discontinuity (i.e., the velocity component tangential to the surface changes from one side to the other side of the surface). We prove that the shear losses on the surface of separation at the die entry are indeed positive, then proceed to calculate the expression for the pressure gradient required to drive the process, and for the pressure distribution on the inner wall of the die.

Further insight into the process of initiation of extrusion is gained by calculating the rates at which mechanical energy is dissipated inside the die working area and upon the die entry surface (Section 4). The results show that an increase in the die angle leads to an increase in the power being dissipated along the die entry and a decrease in the power needed to overcome the resistance of the material to deformation. The analysis shows that the total power required by the process is the same for any die angle. The significance of the result, in view of the known presence of a die-angle effect in continuous forming with conical dies, is discussed in Section 5, which concludes the paper.

2. Plug flow

This type of flow is possible whenever the length of the specimen relative to that of the die is sufficiently small to permit the specimen to be fully confined within the die working area for as long as it is needed by the pseudo-plastic flow of (1) to establish itself, assuming that no buckling or other type of instability occurs.

On condition that the extrusion pressure, p_e, is due to an incompressible fluid being pumped at a constant rate of volumetric flow, Q, the constant of integration B is calculable as before, yielding the expression (3) for the radial velocity. To calculate the other constant of integration, A, we assume a receiving pressure, p_i ($p_i \geqq 0$). This imposes the condition that the stress vector on the front end surface, Σ_i, must be normal to the surface and equal to $-p_i$. The condition is satisfied if Σ_i is spherical all the time and

$$\text{at } r = r_i : \sigma_{rr} = -p_i, \tag{4}$$

where r_i denotes the instantaneous radius of Σ_i and σ is the stress tensor.

In (4), we replace σ_{rr} by $S_{rr} - p$, substitute S_{rr} and p from (1), then solve for A. Substitution of A back into the expression for the hydrostatic pressure in (1) yields

$$p = p_i + 2\sigma_0 \ln \frac{r}{r_i} + \tfrac{2}{3}\sigma_0. \tag{5}$$

The surface, Σ_e, upon which the external pressure, p_e, acts is also spherical if the viscosity of the extruding fluid is low. Moreover, upon Σ_e

$$\text{at } r = r_e : \sigma_{rr} = -p_e, \tag{6}$$

must be satisfied, where r_e denotes the instantaneous radius of Σ_e. We replace σ_{rr} in (6) by $S_{rr} - p$, substitute S_{rr} from (1) and p from (5), then solve for the pressure gradient, $p_e - p_i$, to obtain

$$p_e - p_i = 2\sigma_0 \ln\left(\frac{r_i}{r_e}\right). \tag{7}$$

The pressure distribution on the inner wall of the die, p_w, is given by $-\sigma_{\theta\theta}$, which expressed as $p - S_{\theta\theta}$ is calculable from (1) and (5)

$$p_w = p_i + \sigma_0 + 2\sigma_0 \ln\left(\frac{r}{r_i}\right). \tag{8}$$

The expressions of (7) and (8) are in fact the classical results of Hoffman and Sachs. They are applicable, however, in the strict sense of the word, only to the plug flow since this is the case where Σ_i and Σ_e are indeed spherical and the plastic flow governed by (1) extends truly throughout the region bounded by Σ_i, Σ_e, and the conical surface of the die.

3. Initiation of extrusion

The situation described in this section is likely to occur when the specimen and the die are both sufficiently long for the pseudo-plastic flow governed by (1) to establish itself before any one of the following conditions are violated: 1) there is still a portion left of the undeformed specimen outside the die, and 2) the specimen's front end has not yet reached the die exit (Figure 2).

Assuming that the undeformed portion of the specimen moves as a rigid body with a constant axial velocity, V_s, toward the cone apex, the volumetric rate of flow, Q, is constant and related to V_s by

$$Q = \pi R_s^2 V_s, \tag{9}$$

where R_s denotes the radius of the undeformed portion.

Inside the die working area, the material is assumed to have reached the pseudo-steady state of (1) and, consequently, its radial velocity is given by (3). As before, we denote by p_i, $p_i \geqq 0$, the receiving pressure. The same kind of considerations as those applied for the plug flow lead to the conclusion that the front end surface, Σ_i, is spherical; moreover, the boundary condition at Σ_i is satisfied, when the hydrostatic pressure is calculated from (5) as before.

Although neither the velocity field nor the stress tensor differ here from their counterpart for the plug flow, the two situations are not identical since the extent of the region confining the plastic flow is not the same. In plug flow, the deforming material is confined between two spherical surfaces, Σ_i and Σ_e, and the conical surface of the die inner wall, whereas during the initiation of extrusion, the plastic flow is confined between the spherical surface Σ_i, the conical surface of the die inner wall, and another surface, Σ_s, yet to be calculated, which separates the plastic flow from the rigid motion of the undeformed portion of the specimen, still outside the working area of the die. As we shall see, for positive die angles, Σ_s, is not spherical.

The shape of the surface of separation, Σ_s, must be such that the discontinuity in the velocity is tangential to the surface. We express the shape of Σ_s by the equation

$$F(r, \theta) \equiv r - s(\theta) = 0, \tag{10}$$

where $s(\theta)$ is an unknown function of θ.

The gradient of F, whose components are

$$(\text{grad } F)_r = 1, \ (\text{grad } F)_\theta = -\frac{1}{s}\frac{ds}{d\theta}, \tag{11}$$

is normal to the surface Σ_s. Since the vector representing the difference between the velocity on the two sides of Σ_s:

$$\Delta v_r = (-V_s \cos\theta) - \frac{(-1)Q}{4\pi r^2 \sin^2(\alpha/2)}, \tag{12}$$

$$\Delta v_\theta = V_s \sin\theta,$$

must be tangential to Σ_s, its scalar product with the gradient of F is zero.

$$\Delta v_r(\text{grad } F)_r + \Delta v_\theta(\text{grad } F)_\theta = 0. \tag{13}$$

From (9), (11), (12) and (13), we derive

$$s^2 \cos\theta + ss' \sin\theta = r_s^2 \cos^2(\alpha/2), \tag{14}$$

which is the differential equation to be satisfied by the shape function $s(\theta)$. In (14), s' denotes $ds/d\theta$ and r_s has the value, $r_s = R_s \operatorname{cosec} \alpha$ (see Figure 2).

The boundary condition applicable to (14) is

$$\text{at } \theta = \alpha : s(\theta) = r_s, \tag{15}$$

since the geometry of the problem imposes the condition that all points on the circle ($r = r_s$, $\theta = \alpha$, ϕ)—which is the intersection of the conical surface of the die and the cylindrical surface of the undeformed specimen—are to be on the surface of separation, Σ_s.

The solution to (14) which satisfies (15) is

$$s = r_s \frac{\cos(\alpha/2)}{\cos(\theta/2)}. \tag{16}$$

Substitution of (16) into (10) leads to the following representation for the surface of separation, Σ_s:

$$\text{at } \Sigma_s : r = r_s \frac{\cos(\alpha/2)}{\cos(\theta/2)}, \quad 0 \leqq \theta \leqq \alpha. \tag{17}$$

To be acceptable, the solution so far must pass one more test: the tangential component of the force exercised across each element of Σ_s by the material in rigid motion upon the material in plastic flow must have the same direction as the velocity of the former relative to the latter. In other words, we have to verify that the shear along Σ_s is indeed associated with dissipation of mechanical power.

The velocity of the undeformed material on one side of Σ_s relative to that of the deforming material on the other side is the difference between the absolute velocity (i.e., the velocity relative to the stationary die) of the former and the latter; consequently, the components of the relative velocity are Δv_r and Δv_θ, as calculated in (12).

Considering that the traction components of the stress tensor in (1) must be continuous across Σ_s, the force, ∂T, exercised on a differential element, $\partial\Sigma_s$, by the undeformed material upon the material flowing plastically is calculable in terms of t, the stress vector of the plastic flow:

$$\partial T_r = t_r \partial\Sigma_s, \quad \partial T_\theta = t_\theta \partial\Sigma_s. \tag{18}$$

Accounting only for the non-zero components of the stress tensor, we have: $t_r = n_r\sigma_{rr}$ and $t_\theta = n_\theta\sigma_{\theta\theta}$, where n_r and n_θ are the components of the outward unit normal to Σ_s (i.e., the unit normal directed away from the region confining the plastic flow). The components, n_r and n_θ, are equal to the components of the gradient of F, divided by the magnitude of the gradient. From (11)

$$n_r = \frac{s}{(s^2 + s'^2)^{\frac{1}{2}}}, \quad n_\theta = \frac{(-1)s'}{(s^2 + s'^2)^{\frac{1}{2}}}. \tag{19}$$

With the exception of $\partial\Sigma_s$, everything is now available for calculating ∂T. The element of surface, $\partial\Sigma_s$, $\partial\Sigma_s = 2\pi r \sin\theta[(r d\theta)^2 + (dr)^2]^{\frac{1}{2}}$, is, however, simply calculated by means of equation (10), which permits the replacement of r by s, and of dr by $s' d\theta$. Finally, (16) is used to express s in terms of θ.

Since the relative velocity Δv, has no component normal to Σ_s, testing that the tangential components of ∂T and Δv have the same direction is equivalent to showing that the scalar product of the two vectors is non-negative:

$$\Delta v_r \partial T_r + \Delta v_\theta \partial T_\theta \geqq 0. \tag{20}$$

Incidently, the left hand side expression in (20) is the local rate, $\partial\dot{w}_s$, at which mechanical energy is dissipated per unit time—the shear losses—upon a differential element $\partial\Sigma_s$.

All the variables in (20) have now been calculated in terms of the fields in (1). The use of (1), (3), (5), (9), (12), (16) and (19) in calculating the left hand side of (20), which is $\partial\dot{w}_s$, leads to a positive expression, and proves the correctness of (20).

$$\partial\dot{w}_s = \frac{Q\sigma_0 \sin\theta}{2\sin^2(\alpha/2)} \tan^2(\theta/2) d\theta. \tag{21}$$

To show the importance of the proof of positive shear losses on a surface of velocity discontinuity, let us assume for a moment that the analysis is extended to include the possibility of another surface of separation at the die exit, Σ_s^*, in an attempt to provide a solution to the plastic flow in continuous extrusion. A step-by-step repetition of the previous arguments produces an equation for Σ_s^* similar to equation (17). Moreover, the whole proof of the positive shear losses follows the previous proof (for Σ_s) with one exception only, which is, however, fatal to the success of the proof: the outward unit normal to Σ_s^*

points now in the opposite direction of the gradient of the function representing the surface Σ_s^*; this change in the sign of the shear without a corresponding change in the sign of the relative velocity leads to an expression for the local rate of shear losses, $\partial \dot{w}_s^*$, which is the same as in (21) but with a negative sign.

With the success of the proof of admissibility of Σ_s, the solution is complete and can now be used to calculate the pressure gradient required to drive the process, and the distribution of pressure on the inner wall of the die.

The extrusion pressure, p_e, is calculable from the balance of the axial components of all forces acting on the undeformed specimen (we have selected the positive direction to be that of the flow):

$$\pi R_s^2 p_e + \int_{\Sigma_s} (\partial T_r \cos\theta - \partial T_\theta \sin\theta) = 0. \tag{22}$$

Note that in (22) we have used the continuity of the traction components of the stress tensor on Σ_s, in order to express the force acting upon the undeformed specimen across Σ_s by a force equal in magnitude and opposing ∂T, which acts upon the material in the plastic flow across the same surface.

We calculate the components of ∂T from (18), (19), (1), and (5), then substitute them into (22) and solve it for p_e, to obtain

$$p_e = p_i + 2\sigma_0 \ln\left(\frac{R_s}{R_i}\right), \tag{23}$$

where R_i, $R_i = r_i \sin\alpha$, is the radius of the die at the cross-section where the specimen's front end is located at the instance of time considered.

The pressure gradient required by the process is, therefore,

$$p_e - p_i = 2\sigma_0 \ln\left(\frac{R_s}{R_i}\right). \tag{24}$$

Note that in (24), the radius R_i only is time dependent whereas in the similar equation for the plug flow—equation (7)—both R_e and R_i are time dependent. This is a direct result of the difference which exists in the extent of the region confining the two types of flow. The difference does not affect, however, the stress component, $\sigma_{\theta\theta}$; consequently, the pressure distribution on the die inner wall is that of equation (8).

Before closing the discussion of this section, we like to point out an important consequence of equation (24): the pressure gradient required during the initial phase of extrusion does not depend on the die angle. A detailed discussion of this result follows the power analysis of the next section.

4. Power analysis

The equations for ideal plasticity are so complex that exact analytical solutions are very rare. One of the methods extensively used for obtaining limited analytical information is the upper-bound approach (see Avitzur [3] for a detailed exposition of the method and its application to a variety of metal forming problems). Since the dissipation of power is the main concern of the upper-bound approach, the analysis of this section provides not only a better understanding of the solution obtained, but also a representation of the solution in terms which are directly comparable with the results obtainable by the application of the upper-bound approach.

There are three mechanisms of power losses: 1) the deformation of the material in plastic flow, 2) the shear along surfaces of velocity discontinuity, and 3) friction on the inner wall of the die. With the exception of the friction mechanism, both other mechanisms are present here.

Let $\dot{w}_i$ denote the rate of mechanical energy dissipated per unit volume and unit time, internally, by the stress tensor, σ, acting in conjunction with the rate of strain tensor, $\dot{\varepsilon}$. Accounting only for the non-zero components of σ,

$$\dot{w}_i = \sigma_{rr}\dot{\varepsilon}_{rr} + \sigma_{\theta\theta}\dot{\varepsilon}_{\theta\theta} + \sigma_{\phi\phi}\dot{\varepsilon}_{\phi\phi}. \tag{25}$$

We calculate the rate of strain from the non-zero radial component of the velocity field, v_r, given by (3).

$$\dot{\varepsilon}_{rr} = -\tfrac{1}{2}\dot{\varepsilon}_{\theta\theta} = -\tfrac{1}{2}\dot{\varepsilon}_{\phi\phi} = \frac{Q}{2\pi r^3 \sin^2(\alpha/2)}. \tag{26}$$

Then, use (1), (5), and (26) to obtain an expression for $\dot{w}_i$ from (25). Finally, we integrate over the entire volume of the plastic flow to obtain the total power, $\dot{W}_i$, dissipated internally by the plastic flow:

$$\dot{W}_i = \int_0^{\alpha}\int_{r_i}^{s(\theta)} \frac{Q\sigma_0 \sin\theta}{r \sin^2(\alpha/2)}\, dr\, d\theta. \tag{27}$$

Integration over r, then the use of (16), to express $s(\theta)$, and integration over θ, yield:

$$\dot{W}_i = Q\sigma_0\left\{2\ln\left(\frac{R_s}{R_i}\right) - \left[\frac{2\ln\sec(\alpha/2)}{\sin^2(\alpha/2)} - 1\right]\right\}. \tag{28}$$

Note that an increase in α decreases the value of $\dot{W}_i$.

The shear losses, $\dot{W}_s$, are calculated by summing up $\partial\dot{w}_s$ along Σ_s. From (21)

$$\dot{W}_s = \int_0^{\alpha} \frac{Q\sigma_0 \sin\theta}{2\sin^2(\alpha/2)} \tan^2\frac{\theta}{2}\, d\theta. \tag{29}$$

Integrations yield:

$$\dot{W}_s = Q\sigma_0\left[\frac{2\ln\sec(\alpha/2)}{\sin^2(\alpha/2)} - 1\right]. \tag{30}$$

Note that $\dot{W}_s$ increases with an increase in α.

The total power dissipated by the process, J, is the sum of $\dot{W}_i$ and $\dot{W}_s$:

$$J = 2Q\sigma_0 \ln\left(\frac{R_s}{R_i}\right). \tag{31}$$

As expected, J equals the product $(p_e - p_i)Q\sigma_0$, which is the power provided externally to the process, and does not depend on the die angle. Thus when large die angles are used, in spite of the increase in the shear losses upon the surface of separation, the total power dissipated in the initial stage of the extrusion process does not depend on the die angle: this is because the power dissipated internally is decreased by an amount equal to the increase in shear losses, as a result of changes in Σ_s which decrease the volume of the region confining the plastic flow.

5. Die-angle effect

It is well known that the driving force required for metal forming through a conical die with large die angle increases with the angle of the die. The first estimate of the increase has been calculated by Koerber and Eichinger [3] in their analysis of the wire drawing process. In their view, the force required to support the angle-dependent shear losses at the die entry and exit has to be added to the angle-independent force required for homogeneous deformation (which was already known from the work published in the late 20's and early 30's).

Commenting on Koerber and Eichinger's theory, MacLellan [5] critically notes that "... it [the approach] introduces inconsistency, however, ... the superposition of this supposed additional drawing force must change the distribution of stress on the die wall —precisely how, it is not easy to imagine—in order that the die reaction should continue to balance the net drawing force."

Koerber and Eichinger's work also drew heavy fire from Hill and Tupper [6] who called it "an artificial correction", and dismissed it, apparently, on the grounds that their own analysis of a two-dimensional analog of wire drawing concluded that the increase in the drawing force was accountable by the adjustments needed in the slip-lines when large die angles are used.

In spite of this initial rejection of Koerber and Eichinger's theory, renewed analysis by the upper-bound approach lead to the same conclusion, that the die-angle effect is due to the distortion at the die entry and exit. Since the upper-bound approach does have a theoretical foundation, the result cannot be called anymore "an artificial correction"; moreover, since the method does not permit the calculation of stresses on the inner wall of the die, MacLellan's objection was neatly avoided (although not invalidated!).

Yet, the upper-bound approach gives only an upper bound; therefore, it can be trusted only if there is assurance that the overestimation is somehow uniform for all angles. In other words, the explanation provided by the upper-bound approach as to the cause of the die-angle effect need not be correct if, for some reason or another, the overestimation of the power dissipated is itself dependent on the die angle in a way which tends to show an increase in the power required for large die angles, even when in reality none exists. Indeed, an upper-bound analysis of the plastic flow prevailing during the initiation of extrusion, leads to J^*, an upper bound to J, which depends on the die angle.

$$J^* = Q\sigma_0\left[2f(\alpha)\ln\left(\frac{R_s}{R_i}\right) + \frac{1}{3}\left(\frac{\alpha}{\sin^2\alpha} - \cot\alpha\right)\right]. \tag{32}$$

Equation (32) results if the velocity field of reference [3]:

$$v_r^* = -V_s(r_s/r)^2\cos\theta,\ \ v_\theta = v_\phi = 0, \tag{33}$$

is adopted for the analysis (the function $f(\alpha)$ in (32) increases slowly with the angle of the die but is practically 1 for small angles [3]). Similar die-dependent upper-bounds are likely to result for many other selections of the velocity field.

Any statements which concern the die-angle effect and which are based on the results of this paper cannot be definitive since the problem solved here is not that of continuous forming. Yet, the results of this paper cast, in our view at least, a shadow of doubt on the degree of trust accorded to the explanation provided by the upper-bound approach that the distortion at both, die entry and exit, account for the die-angle effect.

Another case can be made that the solution here is an indication that no inconsistency of the type noted in MacLellan's criticism of Koerber and Eichinger's work need appear when shear losses due to velocity discontinuity are properly included in the analysis of the continuous forming process. Our solution to the pseudo-steady plastic flow necessitated the inclusion of a surface of velocity discontinuity, yet the total force on the die—due to p_w—exactly balances the force due to the pressure differential. This is easily verified by integrating p_w in (8) along the die wall, between $r = r_i$ and $r = r_s$.

Finally, a case can also be made that the absence of a die angle effect in the initial stages of extrusion is a good indication that the effect is due to the yet unknown plastic flow in the region of the die exit, where—as we have seen—a simple surface of velocity discontinuity is not an acceptable description of the phenomena, since it implies negative shear losses.

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